



Axiomatic K-theory for C^* -algebras

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Preface

In *Part I* we present an axiomatic frame in which many results of the K-theory for C*-algebras are proved. In *Part II* we construct an example for this axiomatic theory, which generalizes the classical theory for C*-algebras. This last theory starts by associating to each C*-algebra F the C*-algebras of square matrices with entries in F . Every such C*-algebra of square matrices can be obtained as the projective representation of a certain group with respect to a Schur function for this group with values in \mathbf{C} (Definition 5.0.1). The above mentioned generalization consists in replacing this Schur function by an arbitrary Schur function which satisfies some axiomatic conditions. Moreover this Schur function can take its values in a commutative unital C*-algebra E instead of \mathbf{C} . In this case this K-theory does not apply to the category of C*-algebras, but to the category of E -C*-algebras (Definition 1.1.1), which are C*-algebras endowed with a supplementary structure (every C*-algebra can be endowed with such a supplementary structure (Proposition 1.1.3)). Up to some definitions and notation Part II is independent of *Part I*.

In general we use the notation and the terminology of [1]. In the sequel we give a list of notation used in this book.

- 1) \mathbf{C} (respectively \mathbf{R}) denotes the field of complex (respectively real) numbers, \mathbf{N} denotes the set of natural numbers ($0 \notin \mathbf{N}$), $\mathbf{N}^* := \mathbf{N} \cup \{0\}$, \mathbf{Z} denotes the group of integers, and for every $n \in \mathbf{N}^*$ we put $\mathbf{N}_n := \{k \in \mathbf{N} \mid k \leq n\}$ and $\mathbf{Z}_n := \mathbf{Z}/(n\mathbf{Z})$.
- 2) For every set A , $\text{Card } A$ denotes the cardinal number of A and id_A denotes the identity map of A . If x is a map defined on A and B is a subset of A then $x|_B$ denotes the restriction of x to B .
- 3) Let $(\Omega_j)_{j \in J}$ be a family of topological spaces and let Ω be the disjoint union of this family. The topological sum of the family $(\Omega_j)_{j \in J}$ is the topological space obtained by endowing Ω with the topology $\{U \subset \Omega \mid j \in J \Rightarrow U \cap \Omega_j \text{ is an open set of } \Omega_j\}$.
- 4) If Ω is a topological space and G is a C*-algebra then $\mathcal{C}(\Omega, G)$ denotes the C*-algebra of continuous bounded maps of Ω into G (endowed with the supremum norm). If Ω is a locally compact space then $\mathcal{C}_0(\Omega, G)$ denotes the C*-algebra of continuous maps of Ω into G vanishing at the infinity.
- 5) \odot denotes the algebraic tensor product of vector spaces.
- 6) \approx means isomorphic.

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Throughout this book E denotes a fixed commutative unital C^* -algebra.

