Chapter 5

Multifactorial Design

Multifactorial Design

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Abstract

A researcher may have two or more treatments to handle at a time and these treatments to be handled may or may not interact with each other at certain levels. The effects of these interactions cannot be established with the types of designs such as CRD, RCBD and LSD. The multifactorial design thus helps to establish whether the variations between each type of factor and their interactions are significantly different or not. This paper thus explains how the multifactorial design is used to establish whether there are any significant differences between the treatment types and their interactions.

Keywords

Treatments, Factorial, Factor, Interaction, Variations

5.1 Introduction

Designs like CRD, RCBD and LSD have been used variously under appropriate situations for the design and analysis of experiments. In every experiment, a researcher is bound to handle one or more treatments – that which is expected to be subjected to an experimental material or unit or plot. A factor is defined as a basic treatment and in terms of a multifactorial or factorial designs, a treatment in a sense is a combination of two or more levels of factors. This means that in considering two levels of factors or treatments like the amount of sugar used in the preparation of porridge (A) – half cup of sugar (A_1) and two cups of sugar (A_2); and the quantity of porridge prepared (Q) – twenty (20) litres (Q_1) and twenty-five (25) litres (Q_2) prepared. Thus in handling of factors and treatments at different levels, the researcher would be faced with how to deal or cope with multiple factors. However it must be noted that CRD, RCBD and LSD can be designed and contained in multifactorial designs. It is the appropriate design used in handling treatments with different levels in order to ascertain or establish whether there exist significant differences between the various factor levels or treatment levels as well as the interactions between the treatments.

The factorial experimental designs are more complicated because they are used for designing and analyzing many factors which can be observed on large experimental units and thus not suitable for designing simple experiments. It allows for greater precision when estimating the overall effects of factors as it helps to expose what is referred to as hidden replications. For instance when studying two factors say A and B with two levels A_1, A_2 and B_1, B_2 respectively, the researcher would observe the normal required plots such as A, B and AB and extra plots or units. In all, the researcher would observe these plots or units: A_1B_1, A_1B_2, A_2B_1, A_2B_2, (A_3B_3, this represents interaction between AB).

It must however be noted that the objectives of factorial experimental designs include testing of the main effects or factors and their interactions, and not the treatments. Hence the treatment Mean Squares (MSQ) are not found.

To consider a CRD, let us assume a researcher wants to study the time it takes for the same quantity of milk from cow (A1) and soyabean (A2) to ferment into yoghurt under two temperature conditions B1 and B2. When this experiment is replicated four times, it can be represented below:

5.2 CRD

REP	A1		A2	
	B1	B2	B1	B2
1	5.4	5.3	5.1	4.7
2	4.6	5.6	4.5	4.9
3	4.1	4.8	5.6	5.2
4	3.9	4.8	4.5	5.0

The first step in the analysis is to put forth the hypothesis:

Before analyzing the data, there is the need to put forward the following hypotheses

H_o: $\mu_{A1B1} = \mu_{A1B2} = \mu_{A2B2} = \mu_{A2B2}$

H_1: $\mu_{A1B1} \neq \mu_{A1B2} = \mu_{A2B2} = \mu_{A2B2}$

H_o: All the means of the treatments are equal

H_A: At least one of the means of the treatments is different

H_o: $\mu_{A1} = \mu_{A2}$

H_1: $\mu_{A1} \neq \mu_{A2}$

H_o: The means of the 2 levels of A are equal

H_A: The means of the 2 levels of A are unequal

H_o: $\mu_{B1} = \mu_{B2}$

H_1: $\mu_{B1} \neq \mu_{B2}$

H_o: The means of the 2 levels of B are equal

H_A: The means of the 2 levels of B are unequal

H_o: $\mu_{A \times B} = \mu_{A \times B}$

H_1: $\mu_{A \times B} \neq \mu_{A \times B}$

H_o: The effects of the interaction of two factors at the 2 levels are the same

H_A: The effects of the interaction of two factors at the 2 levels are different

	A_1		A_2		
REP	B_1	B_2	B_1	B_2	
1	5.4	5.3	5.1	4.7	20.5
2	4.6	5.6	4.5	4.9	19.6
3	4.1	4.8	5.6	5.2	19.7
4	3.9	4.8	4.5	5	18.2
	18	20.5	19.7	19.8	78

$$Tt_{A_1B_1} = \sum(5.4 + 4.6 + \cdots + 3.9) = 18$$

$$Tt_{A_1B_2} = \sum(5.3 + 5.6 + \cdots + 4.8) = 20.5$$

$$Tt_{A_2B_1} = \sum(5.1 + 4.5 + \cdots + 4.5) = 19.7$$

$$Tt_{A_2B_2} = \sum(4.7 + 4.9 + \cdots + 5) = 19.8$$

$$Tt_T = \sum\left(Tt_{A_1B_1} + Tt_{A_1B_2} + Tt_{A_2B_2} + Tt_{A_2B_2}\right) = 78$$

The second step is calculating of the Correction Factor:

$$CF = \frac{\sum(Tt_T)^2}{(r\ x\ t)} = \frac{(78)^2}{4\ x\ 4} = \frac{(78)^2}{16} = \frac{6084}{16} = 380.25$$

The third step is calculating of the Total Sum of Squares:

$$TSS =$$
$$\sum((5.4 + (5.3)^2 + (5.1)^2 + (4.7)^2 + (4.6)^2 + (5.6)^2 \ldots + (5)^2) - 380.25 = 3.63$$

The fourth step is calculating of the Treatment Sum of Squares:

$$TrtSS = \sum\left(\frac{(Tt_{A_1B_1})^2 + (Tt_{A_1B_2})^2 + (Tt_{A_2B_2})^2 + (Tt_{A_2B_2})^2}{4}\right) - 380.25 = 0.85$$

$$TrtSS = \sum\left(\frac{(18)^2 + (20.5)^2 + (19.7)^2 + (19.8)^2}{4}\right) - 380.25 = 0.85$$

$$TrtSS = \sum\left(\frac{1524.38}{4}\right) - 380.25 = 0.85$$

$$TrtSS = 381.10 - 380.25 = 0.85$$

The fifth step is calculating of the Factor "A" Sum of Squares (ASS):

$$A = A_1 + A_2$$
$$A_1 = 18 + 20.5 = 38.5$$
$$A_2 = 19.7 + 19.8 = 39.5$$
$$ASS = \sum\left(\frac{(A_1)^2 + (A_2)^2}{r\ x\ n}\right) - CF$$
$$ASS = \sum\left(\frac{(38.5)^2 + (39.5)^2}{4\ x\ 2}\right) - 380.25$$
$$ASS = \sum\left(\frac{1482.25 + 1560.25}{8}\right) - 380.25$$
$$ASS = \sum\left(\frac{3042.5}{8}\right) - 380.25$$

$$ASS = \sum 380.31 - 380.25 = 0.063$$

The sixth step is calculating of the Factor "B" Sum of Squares (BSS):

$$B = B_1 + B_2$$

$$B_1 = 18 + 19.7 = 37.7$$

$$B_2 = 20.5 + 19.8 = 40.3$$

$$BSS = \sum (\frac{(B_1)^2 + (B_2)^2}{r \ x \ n}) - CF$$

$$BSS = \sum (\frac{(37.7)^2 + (40.3)^2}{4 \ x \ 2}) - 380.25$$

$$BSS = \sum (\frac{1421.29 + 1624.09}{8}) - 380.25$$

$$BSS = \sum (\frac{3045.38}{8}) - 380.25$$

$$BSS = \sum 380.67 - 380.25 = 0.42$$

The seventh step is calculating of the interaction between "A x B" Sum of Squares (ABSS):

$$ABSS = TrtSS - (ASS + BSS)$$

$$ABSS = 0.85 - (0.063 + 0.42)$$

$$ABSS = 0.85 - 0.48 = 0.37$$

The eighth step is calculating of the Error Sum of Squares (ESS):

$$ESS = TSS - TrtSS = 3.63 - 0.85 = 2.78$$

Tt_T=Treatment Total, ASS=Factor "A" Sum of Squares, TSS=Total Sum of Squares, BSS=Factor "B" Sum of Squares, TrtSS=Treatment Sum of Squares, ABSS=Interaction between Factors "A x B" Sum of Squares, ESS=Error Sum of Squares, CF=Correction Factor, n = number of levels of factors, r = number of replication.

The ninth step looking up the F-critical table to the F-critical values:

In looking for the critical value, the degree of the variable of the variable whose critical value is being read from the table is located on the outer most row and matched with the error degree of freedom on the outer most column on the left of the table. Following the given procedure, the following readings would be obtained from the 5% and 1% F-critical table:

Treatment: F crit at df 3, 9 for 5% = 3.86; F crit at df 3, 9 for 1% = 6.99

Factor A: F crit at df 1, 9 for 5% = 5.12; F crit at df 1, 9 for 1% = 10.56

Factor B: F crit at df 1, 9 for 5% = 5.12; F crit at df 1, 9 for 1% = 10.56

Interaction (A x B): F crit at df 1, 9 for 5% = 5.12; F crit at df 1, 9 for 1% = 10.56.

The tenth step is completing the ANOVA table:

ANOVA TABLE						
Sources of Variation	*df*	*SS*	*MS*	*Fcal.*	*Fcrit. (5%)*	*Fcrit (1%)*
Treatment	4 - 1= 3	0.85	0.28	0.92	3.86	6.99
Factor A	2 - 1 = 1	0.063	0.06	0.20	5.12	10.56
Factor B	2 - 1 = 1	0.42	0.42	1.36	5.12	10.56
Interaction (A x B)	2 − 1 =1	0.37	0.37	1.20	5.12	10.56
Error	15 − 6 = 9	2.78	0.31			
Total	16 − 1=15	3.63				

The eleventh step is making the decision or conclusion:

Decision on Treatment

$$Fcal\ (0.92) < 3.86, Fcrit.\ or\ tab.\ (5\%) at\ 3, 9$$

$$Fcal\ (\ 0.92) < 6.99, Fcrit.\ or\ tab.\ (1\%) at\ 3, 9$$

Since *Fcal < Fcrit. or tab. at both 1% and 5% levels of significance*, it can be concluded that there are no significant differences between the various treatments and then fail to reject the null hypothesis.

Decision of Factor A

$$Fcal\ (0.20) < 5.12, Fcrit.or\ tab.(5\%)at\ 1,9$$
$$Fcal\ (\ 0.20) < 10.56, Fcrit.or\ tab.(1\%)at\ 1,9$$

Fcal < Fcrit. or tab. at both 1% and 5% levels of significance, hence there are no significant differences between the two different levels of factor A.

Decision of Factor B

$$Fcal\ (1.36) < 5.12, Fcrit.or\ tab.(5\%)at\ 1,9$$
$$Fcal\ (\ 1.36) < 10.56, Fcrit.or\ tab.(1\%)at\ 1,9$$

Fcal < Fcrit. or tab. at both 1% and 5% levels of significance, hence there are no significant differences between the two different levels of factor B.

Decision on the interactions (A x B)

$$Fcal\ (1.20) < 5.12, Fcrit.or\ tab.(5\%)at\ 1,9$$
$$Fcal\ (\ 1.20) < 10.56, Fcrit.or\ tab.(1\%)at\ 1,9$$

Fcal < Fcrit. or tab. at both 1% and 5% levels of significance, hence the two levels of factor A do not vary significantly at the two levels of B when applied together.

5.3 RCBD

With regards to RCBD, the researcher can study the time it takes for the same quantity of milk from cow (A_1) and (A_2) to ferment into yoghurt under two

temperature conditions B_1 and B_2 blocked under the mode of storage: block 1(fridge), block 2 (air condition) and block 3 (normal room condition).

	A_1		A_2	
BLK	B_1	B_2	B_1	B_2
1	5.4	5.3	5.1	4.7
2	4.6	5.6	4.5	4.9
3	4.1	4.8	5.6	5.2
4	3.9	4.8	4.5	5.0

The first step in the analysis is to put forth the hypothesis:

For RCBD the following hypotheses can be put forth:

H_o: $\mu BLK_1 = \mu BLK_2 = \mu BLK_3 = \mu BLK_4$

H_1: $\mu BLK_1 \neq \mu BLK_2 = \mu BLK_3 = \mu BLK_4$

H_o: All blocks have equal treatment means

H_A: At least one block's treatment means differ from the others

H_o: $\mu A_1 B_1 = \mu A_1 B_2 = \mu_{A2B2} = \mu_{A2B2}$

H_1: $\mu_{A1B1} \neq \mu_{A1B2} = \mu_{A2B2} = \mu_{A2B2}$

H_o: All the means of the treatments are equal

H_A: At least one of the means of the treatments is different

H_o: $\mu A_1 = \mu A_2$

H_1: $\mu_{A1} \neq \mu_{A2}$

H_o: The means of the 2 levels of A are equal

H$_A$: The means of the 2 levels of A are unequal

H$_o$: $\mu_{B1} = \mu_{B2}$

H$_1$: $\mu_{B1} \neq \mu_{B2}$

H$_o$: The means of the 2 levels of B are equal

H$_A$: The means of the 2 levels of B are unequal

H$_o$: $\mu_{A \times B} = \mu_{A \times B}$

H$_1$: $\mu_{A \times B} \neq \mu_{A \times B}$

H$_o$: The effects of the interaction of two factors at the 2 levels are the same

H$_A$: The effects of the interaction of two factors at the 2 levels are different

	A$_1$		A$_2$		
BLK	**B$_1$**	**B$_2$**	**B$_1$**	**B$_2$**	
1	5.4	5.3	5.1	4.7	20.5
2	4.6	5.6	4.5	4.9	19.6
3	4.1	4.8	5.6	5.2	19.7
4	3.9	4.8	4.5	5	18.2
	18	20.5	19.7	19.8	78

$$Tt_{A_1B_1} = \sum(5.4 + 4.6 + \cdots + 3.9) = 18$$

$$Tt_{A_1B_2} = \sum(5.3 + 5.6 + \cdots + 4.8) = 20.5$$

$$Tt_{A_2B_1} = \sum(5.1 + 4.5 + \cdots + 4.5) = 19.7$$

$$Tt_{A_2B_2} = \sum(4.7 + 4.9 + \cdots + 5) = 19.8$$

$$Tt_T = \sum\left(Tt_{A_1B_1} + Tt_{A_1B_2} + Tt_{A_2B_2} + Tt_{A_2B_2}\right) = 78$$

The second step is calculating of the Correction Factor:

$$CF = \frac{\sum(Tt_T)^2}{t} = \frac{\sum(Tt_T)^2}{4\ x\ 2\ x\ 2} = \frac{(78)^2}{4\ x\ 4} = \frac{(78)^2}{16} = \frac{6084}{16} = 380.25$$

where t=number of blocks x 2 levels of A x 2 levels of B=4 x 2 x 2

The third step is calculating of the Total Sum of Squares:

$$TSS =$$
$$\sum((5.4 + (5.3)^2 + (5.1)^2 + (4.7)^2 +\ (4.6)^2 + (5.6)^2 \dots . +(5)^2) - 380.25 = 3.63$$

The fourth step is calculating of the Treatment Sum of Squares:

$$TrtSS = \sum\left(\frac{(Tt_{A_1B_1})^2 + (Tt_{A_1B_2})^2 + (Tt_{A_2B_2})^2 + (Tt_{A_2B_2})^2}{4}\right) - 380.25 = 0.85$$

$$TrtSS = \sum\left(\frac{(18)^2 + (20.5)^2 + (19.7)^2 + (19.8)^2}{4}\right) - 380.25 = 0.85$$

$$TrtSS = \sum\left(\frac{1524.38}{4}\right) - 380.25 = 0.85$$

$$TrtSS = 381.10 - 380.25 = 0.85$$

The fifth step is calculating of the Factor "A" Sum of Squares (ASS):

$$A = A_1 + A_2$$
$$A_1 = 18 + 20.5 = 38.5$$
$$A_2 = 19.7 + 19.8 = 39.5$$
$$ASS = \sum\left(\frac{(A_1)^2 + (A_2)^2}{r\ x\ n}\right) - CF$$
$$ASS = \sum\left(\frac{(38.5)^2 + (39.5)^2}{4\ x\ 2}\right) - 380.25$$
$$ASS = \sum\left(\frac{1482.25 + 1560.25}{8}\right) - 380.25$$
$$ASS = \sum\left(\frac{3042.5}{8}\right) - 380.25$$

$$ASS = \sum 380.31 - 380.25 = 0.063$$

The sixth step is calculating of the Factor "A" Sum of Squares (ASS):

$$A = A_1 + A_2$$

$$A_1 = 18 + 20.5 = 38.5$$

$$A_2 = 19.7 + 19.8 = 39.5$$

$$ASS = \sum \left(\frac{(A_1)^2 + (A_2)^2}{r \; x \; n}\right) - CF$$

$$ASS = \sum \left(\frac{(38.5)^2 + (39.5)^2}{4 \; x \; 2}\right) - 380.25$$

$$ASS = \sum \left(\frac{1482.25 + 1560.25}{8}\right) - 380.25$$

$$ASS = \sum \left(\frac{3042.5}{8}\right) - 380.25$$

$$ASS = \sum 380.31 - 380.25 = 0.063$$

The seventh step is calculating of the Factor "B" Sum of Squares (BSS):

$$B = B_1 + B_2$$

$$B_1 = 18 + 19.7 = 37.7$$

$$B_2 = 20.5 + 19.8 = 40.3$$

$$BSS = \sum \left(\frac{(B_1)^2 + (B_2)^2}{r \; x \; n}\right) - CF$$

$$BSS = \sum \left(\frac{(37.7)^2 + (40.3)^2}{4 \; x \; 2}\right) - 380.25$$

$$BSS = \sum \left(\frac{1421.29 + 1624.09}{8}\right) - 380.25$$

$$BSS = \sum \left(\frac{3045.38}{8}\right) - 380.25$$

$$BSS = \sum 380.67 - 380.25 = 0.42$$

The eighth step is calculating the Block Sum of Squares (BLKSS):

$$BLKSS = \sum \left(\frac{(BLK1_T)^2 + (BLK2_T)^2 + (BLK3_T)^2 + (BLK4_T)^2}{number \ of \ blocks} \right) - CF$$

$$BLKSS = \sum \left(\frac{(20.5)^2 + (19.6)^2 + (19.7)^2 + (18.2)^2}{4} \right) - 380.25$$

$$BLKSS = \sum \left(\frac{(420.25) + (384.16) + (388.09) + (331.24)}{4} \right) - 380.25$$

$$BLKSS = \sum \left(\frac{(1523.74)}{4} \right) - 380.25$$

$$BLKSS = \sum 380.94 - 380.25 = 0.69$$

The ninth step is calculating of the interaction between "A x B" Sum of Squares (ABSS):

$$ABSS = TrtSS - (ASS + BSS)$$

$$ABSS = 0.85 - (0.063 + 0.42)$$

$$ABSS = 0.85 - 0.48 = 0.37$$

The tenth step is calculating of the Error Sum of Squares (ESS):

$$ESS = TSS - (BLKSS + TrtSS)$$

$$ESS = 3.63 - (0.85 + 0.69)$$

$$ESS = 3.63 - 1.54 = 2.09$$

Tt_T=Treatment Total, ASS=Factor "A" Sum of Squares, TSS=Total Sum of Squares, BSS=Factor "B" Sum of Squares, TrtSS=Treatment Sum of Squares, BLKSS=Block Sum of Squares, ABSS=Interaction between Factors "A x B" Sum of Squares, ESS=Error Sum of Squares, CF=Correction Factor, n=number levels of factors, r = number of replication.

The eleventh step looking up the f-critical table to the f- critical values:

Block: f crit at df 3, 6 for 5% = 4.76; f crit at df 3, 6 for 1% = 9.78

Treatment: F crit at df 3, 6 for 5% = 4.76; F crit at df 3, 6 for 1% = 9.78

Factor A: F crit at df 1, 6 for 5% = 5.99; F crit at df 1, 6 for 1% = 13.75

Factor B: F crit at df 1, 6 for 5% = 5.99; F crit at df 1, 6 for 1% = 13.75

Interaction (A x B): F crit at df 1, 6 for 5% = 5.99; F crit at df 1, 6 for 1% = 13.75.

The twelfth step is completing the ANOVA table:

ANOVA TABLE						
Sources of Variation	*df*	*SS*	*MS*	*Fcal.*	*Fcrit. (5%)*	*Fcrit (1%)*
Block	4 – 1= 3	0.69	0.23	0.66	4.76	9.78
Treatment	4 - 1= 3	0.85	0.28	0.81	4.76	9.78
Factor A	2 - 1 = 1	0.063	0.06	0.18	5.99	13.75
Factor B	2 - 1 = 1	0.42	0.42	1.21	5.99	13.75
Interaction (A x B)	2 – 1 =1	0.37	0.37	1.06	5.99	13.75
Error	15 – 9 = 6	2.09	0.35			
Total	16 - 1=15	3.63				

The thirteenth step is making the decision or conclusion:

Decision on Block

$$Fcal\ (0.66) < 4.76, Fcrit. or\ tab. (5\%) at\ 3, 6$$
$$Fcal\ (\ 0.66) < 9.78, Fcrit. or\ tab. (1\%) at\ 3, 6$$

The F cal < F crit for the block at both 1% and 5% levels of significance, hence the researcher must fail to reject the null hypothesis for the block and conclude that there is no significant difference between the blocks. It there for suggests that there is no justification for blocking and the RCBD is therefore not the appropriate design for analysis of this experiment.

Decision on Treatment

$$Fcal\ (0.81) < 4.76, Fcrit.or\ tab.(5\%)at\ 3,6$$
$$Fcal\ (\ 0.81) < 9.78, Fcrit.or\ tab.(1\%)at\ 3,6$$

Since *Fcal < Fcrit. or tab. at both 1% and 5% levels of significance, it can* be concluded that there are no significant differences between the various treatments and then fail to reject the null hypothesis.

Decision of Factor A

$$Fcal\ (0.18) < 5.99, Fcrit.or\ tab.(5\%)at\ 1,6$$
$$Fcal\ (\ 0.18) < 13.75, Fcrit.or\ tab.(1\%)at\ 1,6$$

Fcal < Fcrit. or tab. at both 1% and 5% levels of significance, hence there are no significant differences between the two different levels of factor A.

Decision of Factor B

$$Fcal\ (1.21) < 5.99, Fcrit.or\ tab.(5\%)at\ 1,6$$
$$Fcal\ (\ 1.21) < 13.75, Fcrit.or\ tab.(1\%)at\ 1,6$$

Fcal < Fcrit. or tab. at both 1% and 5% levels of significance, hence there are no significant differences between the two different levels of factor B.

Decision on the interactions (A x B)

$$Fcal\ (1.06) < 5.99, Fcrit.or\ tab.(5\%)at\ 1,6$$
$$Fcal\ (\ 1.06) < 13.75, Fcrit.or\ tab.(1\%)at\ 1,6$$

Fcal < Fcrit. or tab. at both 1% and 5% levels of significance, hence the two levels of factor A do not vary significantly at the two levels of B when applied together.

5.4 LSD

For the LSD, the same experiment can be used as in the case of CRD and RCBD, but this time around there is no replication and blocking as done respectively under the two mentioned designs.

		A_1		A_2	
		B_1	B_2	B_1	B_2
A_1	B_1	5.4	5.3	5.1	4.7
	B_2	4.6	5.6	4.5	4.9
A_2	B_1	4.1	4.8	5.6	5.2
	B2	3.9	4.8	4.5	5.0

The first step in the analysis is to put forth the hypothesis:

The following hypotheses can be stated for the LSD:

H_o: $\mu COL_1 = \mu COL_2 = \mu COL_3 = \mu COL_4$

H_1: $\mu COL_1 \neq \mu COL_2 = \mu COL_3 = \mu COL_4$

H_o: All columns have equal treatment means

H_A: At least one column's treatment means differ from the others

H_o: $\mu ROW_1 = \mu ROW_2 = \mu ROW_3 = \mu ROW_4$

H_1: $\mu ROW_1 \neq \mu ROW_2 = \mu ROW_3 = \mu ROW_4$

H_o: All rows have equal treatment means

H_A: At least one row's treatment means differ from the others

H_o: $\mu A_1 B_1 = \mu A_1 B_2 = \mu_{A2B2} = \mu_{A2B2}$

H_1: $\mu_{A1B1} \neq \mu_{A1B2} = \mu_{A2B2} = \mu_{A2B2}$

H_o: All the means of the treatments are equal

H_A: At least one of the means of the treatments is different

H_o: $\mu A_1 = \mu A_2$

H_1: $\mu_{A1} \neq \mu_{A2}$

H_o: The means of the 2 levels of A are equal

H_A: The means of the 2 levels of A are unequal

H_o: $\mu_{B1} = \mu_{B2}$

H_1: $\mu_{B1} \neq \mu_{B2}$

H_o: The means of the 2 levels of B are equal

H_A: The means of the 2 levels of B are unequal

H_o: $\mu_{A \times B} = \mu_{A \times B}$

H_1: $\mu_{A \times B} \neq \mu_{A \times B}$

H_o: The effects of the interaction of two factors at the 2 levels are the same

H_A: The effects of the interaction of two factors at the 2 levels are different

		A_1		A_2		
		B_1	B_2	B_1	B_2	
A_1	B_1	5.4	5.3	5.1	4.7	20.5
	B_2	4.6	5.6	4.5	4.9	19.6
A_2	B_1	4.1	4.8	5.6	5.2	19.7
	B_2	3.9	4.8	4.5	5	18.2
		18	20.5	19.7	19.8	78

The second step is calculating of the Correction Factor:

$$CF = \frac{\sum(Tt_T)^2}{t} = \frac{\sum(Tt_T)^2}{4 \; x \; 2 \; x \; 2} = \frac{(78)^2}{4 \; x \; 4} = \frac{(78)^2}{16} = \frac{6084}{16} = 380.25$$

where t=number of blocks x 2 levels of A x 2 levels of B=4 x 2 x 2

The third step is calculating of the Total Sum of Squares:

$$TSS =$$
$$\sum((5.4 + (5.3)^2 + (5.1)^2 + (4.7)^2 + (4.6)^2 + (5.6)^2 \ldots + (5)^2) - 380.25 = 3.63$$

The fourth step is calculating of the Treatment Sum of Squares:

$$TrtSS = \sum\left(\frac{(Tt_{A_1B_1})^2 + (Tt_{A_1B_2})^2 + (Tt_{A_2B_2})^2 + (Tt_{A_2B_2})^2}{4}\right) - 380.25 = 0.85$$

$$TrtSS = \sum\left(\frac{(18)^2 + (20.5)^2 + (19.7)^2 + (19.8)^2}{4}\right) - 380.25 = 0.85$$

$$TrtSS = \sum\left(\frac{1524.38}{4}\right) - 380.25 = 0.85$$

$$TrtSS = 381.10 - 380.25 = 0.85$$

The fifth step is calculating of the Column Sum of Squares (CSS):

$$CSS = \sum\left(\frac{(COL1_T)^2 + (COL2_T)^2 + (COL3_T)^2 + (COL4_T)^2}{number \; of \; columns}\right) - CF$$

$$CSS = \sum\left(\frac{(18)^2 + (20.5)^2 + (19.7)^2 + (19.8)^2}{4}\right) - 380.25$$

$$CSS = \sum\left(\frac{(324) + (420.25) + (388.09) + (392.04)}{4}\right) - 380.25$$

$$CSS = \sum 381.10 - 380.25 = 0.85$$

The sixth step is calculating of the Row Sum of Squares (RSS):

$$RSS = \sum\left(\frac{(ROW1_T)^2 + (ROW2_T)^2 + (ROW3_T)^2 + (ROW4_T)^2}{number \; of \; rows}\right) - CF$$

$$RSS = \sum \left(\frac{(20.5)^2 + (19.6)^2 + (19.7)^2 + (18.2)^2}{4} \right) - 380.25$$

$$RSS = \sum \left(\frac{(420.25) + (384.16) + (388.09) + (331.24)}{4} \right) - 380.25$$

$$RSS = \sum 380.94 - 380.25 = 0.69$$

The seventh step is calculating of the Factor "A" Sum of Squares (ASS):

$$A = A_1 + A_2$$

$$A_1 = 18 + 20.5 = 38.5$$

$$A_2 = 19.7 + 19.8 = 39.5$$

$$ASS = \sum \left(\frac{(A_1)^2 + (A_2)^2}{r \, x \, n} \right) - CF$$

$$ASS = \sum \left(\frac{(38.5)^2 + (39.5)^2}{4 \, x \, 2} \right) - 380.25$$

$$ASS = \sum \left(\frac{1482.25 + 1560.25}{8} \right) - 380.25$$

$$ASS = \sum \left(\frac{3042.5}{8} \right) - 380.25$$

$$ASS = \sum 380.31 - 380.25 = 0.063$$

The eighth step is calculating of the Factor "A" Sum of Squares (ASS):

$$A = A_1 + A_2$$

$$A_1 = 18 + 20.5 = 38.5$$

$$A_2 = 19.7 + 19.8 = 39.5$$

$$ASS = \sum \left(\frac{(A_1)^2 + (A_2)^2}{r \, x \, n} \right) - CF$$

$$ASS = \sum \left(\frac{(38.5)^2 + (39.5)^2}{4 \, x \, 2} \right) - 380.25$$

$$ASS = \sum \left(\frac{1482.25 + 1560.25}{8} \right) - 380.25$$

$$ASS = \sum \frac{3042.5}{8} - 380.25$$

$$ASS = \sum 380.31 - 380.25 = 0.063$$

The ninth step is calculating of the Factor "B" Sum of Squares (BSS):

$$B = B_1 + B_2$$

$$B_1 = 18 + 19.7 = 37.7$$

$$B_2 = 20.5 + 19.8 = 40.3$$

$$BSS = \sum (\frac{(B_1)^2 + (B_2)^2}{r \ x \ n}) - CF$$

$$BSS = \sum (\frac{(37.7)^2 + (40.3)^2}{4 \ x \ 2}) - 380.25$$

$$BSS = \sum (\frac{1421.29 + 1624.09}{8}) - 380.25$$

$$BSS = \sum (\frac{3045.38}{8}) - 380.25$$

$$BSS = \sum 380.67 - 380.25 = 0.42$$

The tenth step is calculating of the interaction between "A x B" Sum of Squares (ABSS):

$$ABSS = TrtSS - (ASS + BSS)$$

$$ABSS = 0.85 - (0.063 + 0.42)$$

$$ABSS = 0.85 - 0.48 = 0.37$$

The eleventh step is calculating of the Error Sum of Squares (ESS):

$$ESS = TSS - (TrtSS + RSS + CSS)$$

$$ESS = 3.63 - (0.85 + 0.69 + 0.85)$$

$$ESS = 3.63 - 2.39 = 1.24$$

Tt_T=Treatment Total, ASS=Factor "A" Sum of Squares, TSS=Total Sum of Squares, BSS=Factor "B" Sum of Squares, TrtSS=Treatment Sum of Squares, BLKSS=Block Sum of Squares, ABSS=Interaction between Factors "A x B" Sum of Squares, ESS=Error Sum of Squares, CF=Correction Factor, n=number levels of factors, r = number of replication.

The twelfth step looking up the F-critical table to the F-critical values:

Column: F crit at df 3, 3 for 5% =9.28; F crit at df 3, 3 for 1% = 29.46

Row: F crit at df 3, 3 for 5% = 9.28; F crit at df 3, 3 for 1% = 29.46

Treatment: F crit at df 3, 3 for 5% = 9.28; F crit at df 3, 3 for 1% = 29.46

Factor A: F crit at df 1, 3 for 5% = 10.13; F crit at df 1, 3 for 1% = 34.12

Factor B: F crit at df 1, 3 for 5% = 10.13; F crit at df 1, 3 for 1% = 34.12

Interaction (A x B): F crit at df 1, 3 for 5% = 10.13; F crit at df 1, 3 for 1% = 34.12.

The thirteenth step is completing the ANOVA table:

Sources of Variation	df	SS	MS	Fcal.	Fcrit. (5%)	Fcrit (1%)
Column	4 – 1= 3	0.69	0.23	0.56	9.28	29.46
Row	4 -1 = 3	0.85	0.28	0.69	9.28	29.46
Treatment	4 - 1= 3	0.85	0.28	0.69	9.28	29.46
Factor A	2 - 1 = 1	0.063	0.06	0.15	10.13	34.12
Factor B	2 - 1 = 1	0.42	0.42	1.02	10.13	34.12
Interaction (A x B)	2 – 1 =1	0.37	0.37	0.90	10.13	34.12
Error	15 – 12 = 3	1.24	0.41			
Total	16 -1=15	3.63				

The fourteenth step is making the decision or conclusion:

Decision on Column

$$Fcal\ (0.56) < 9.28, Fcrit.\ or\ tab.\ (5\%) at\ 3, 3$$
$$Fcal\ (\ 0.56) < 29.46, Fcrit.\ or\ tab.\ (1\%) at\ 3, 3$$

The F cal < F crit for the block at both 1% and 5% levels of significance, hence the researcher must fail to reject the null hypothesis for the column and conclude that there is no significance difference between the columns.

Decision on Row

$$Fcal\ (0.69) < 9.28, Fcrit.\ or\ tab.\ (5\%) at\ 3, 3$$
$$Fcal\ (\ 0.69) < 29.46, Fcrit.\ or\ tab.\ (1\%) at\ 3, 3$$

Fcal < Fcrit. or tab. at both 1% and 5% levels of significance, it can be concluded that there are no significant differences between rows hence the researcher must fail to reject the null hypothesis for the row.

Once it has been established that there are no significant differences between the columns and rows respectively in the design, one can therefore conclude that there is no justification for the use of LSD, it is thus inappropriate for analysis of the experiment in question.

Decision on Treatment

$$Fcal\ (0.69) < 9.28, Fcrit.\ or\ tab.\ (5\%) at\ 3, 3$$
$$Fcal\ (\ 0.69) < 29.46, Fcrit.\ or\ tab.\ (1\%) at\ 3, 3$$

Since *Fcal < Fcrit. or tab. at both 1% and 5% levels of significance,* it can be concluded that there are no significant differences between the various treatments and then fail to reject the null hypothesis for the treatment.

Decision of Factor A

$$Fcal\ (0.15) < 10.13, Fcrit.\ or\ tab.\ (5\%)\ at\ 1, 3$$

$$Fcal\ (\ 0.15) < 34.12, Fcrit.\ or\ tab.\ (1\%)\ at\ 1, 3$$

Fcal < Fcrit. or tab. at both 1% and 5% levels of significance, hence there are no significant differences between the two different levels of factor A.

Decision of Factor B

$$Fcal\ (1.02) < 10.13, Fcrit.\ or\ tab.\ (5\%)\ at\ 1, 3$$

$$Fcal\ (\ 1.02) < 34.12, Fcrit.\ or\ tab.\ (1\%)\ at\ 1, 3$$

Fcal < Fcrit. or tab. at both 1% and 5% levels of significance, hence there are no significant differences between the two different levels of factor B.

Decision on the interactions (A x B)

$$Fcal\ (0.90) < 10.13, Fcrit.\ or\ tab.\ (5\%)\ at\ 1, 3$$

$$Fcal\ (\ 0.90) < 34.12, Fcrit.\ or\ tab.\ (1\%)\ at\ 1, 6$$

Fcal < Fcrit. or tab. at both 1% and 5% levels of significance, hence the two levels of factor A do not vary significantly at the two levels of B when applied together.

Bibliography

[1] Brownlee, K. A. (1960). *Statistical theory and methodology in science and engineering*. New York: Wiley.

[2] Campbell, D., Stanley, J. (1963). *Experimental and quasi-experimental designs for research and teaching*. In Gage (Ed.), Handbook on research on teaching. Chicago: Rand McNally & Co.

[3] Winer, B. J. (1962). *Statistical principles in experimental design*. New York: McGraw-Hill.

[4] Cochran, W. G., Cox, G. M. (1957). *Experimental Designs.* John Willey & Sons, N.Y.

[5] Le Clerg, E. L., Leonard, W. H., Clark A. G. (1966). *Field Plot Technique.* Burgess Pub. Co., Minn., USA.

[6] Daniel, C. (1959). Use of Half-Normal Plots in Interpreting Factorial Two-Level Experiments. *Technometrics* 1:311–341.

[7] Fisher, R. A. (1925). *Statistical Methods for Research Workers*. Oliver and Boyd, Edinburgh.

[8] Ju, H. L., Lucas, J. M. (2002). L*k* Factorial Experiments with Hard-To-Change and Easy-To-Change Factors. *Journal of Quality Technology* 34:411–421.

[9] Montgomery, D. C. (2008). *Design and Analysis of Experiments*. 7th ed. John Wiley & Sons. New York, NY.

[10] Taguchi, G. (1987). *System of Experimental Design*. Kraus International Publications, White Plains, NY.

[11] Webb, D. F., Lucas, J. M., Borkowski, J. J. (2004). Factorial Experiments when Factor Levels Are Not Necessarily Reset. *Journal of Quality Technology* 36:1–11.

[12] Yates, F. (1935). Complex Experiments with Discussion. *Journal of the Royal Statistical Society, Series B* 2:181–223.